The Open University

S102PRP/S510 MODULE 2 INTO SCIENCE

MODULE 2 OBSERVING AND MEASURING

STUDY GUIDE

The main aim of this Module is to introduce you to the skills of observing and measuring. These skills form the basis of all practical work and it is upon practical work and experiments that science depends. When making measurements a number of things need to be borne in mind: what units of measurement to use, how accurate the measurements need to be, and how to record and present the results. One way of presenting results is in tables and this Module shows you how to do this.

Having learned how to make some measurements and present the results, you will be asked to make some measurements of your own. You will need the following equipment to hand:

a 300 millimetre (i.e. a 30 centimetre) ruler

a length of cotton about 30 centimetres long

access to a small tree or bush from which you can pick ten leaves.

In Module 1, you were introduced to one approach to the study of a text. First skim through the Module to get a general idea of what it covers. This should include reading the contents list, looking at the exercises and reading the Overview. Then read it through much more thoroughly (perhaps covering one section during a given session), studying each paragraph carefully and marking those sentences or phrases that you think are important. In addition you may wish to annotate the margins of your text with any questions or problems you might have, as well as with notes designed to summarize the text or help you follow the discussion. Try to complete the exercises because these will help you to think more carefully about the text.

When you have finished studying the Module, check that you have achieved reasonable competence in the skills listed in the Overview (Section 8); these are tested by SAQs.

It is difficult to estimate the amount of time you need to allow for studying this Module, but on average you should expect to spend about three hours. This includes half an hour for picking and measuring the leaves and a further half an hour for handling your results. Don't worry if you take longer than this.

1 MAKING MEASUREMENTS

1.1 THE SCIENTIFIC APPROACH

The pursuit of scientific knowledge begins with the activities: observing, measuring and recording. Records of observations and measurements are usually referred to as 'data'. The next stage is to look for patterns in the collected data. Scientists then try to explain these patterns, and to make predictions about similar situations. This in turn leads to the development of theories, that is speculative thoughts or ideas. It is important to test whether the predictions hold true, and this can only be achieved by further practical work or experiment.

If experiment confirms the predictions, then the theory or idea is accepted until further measurements and predictions advance the idea further. If, however, practical work does not support the predictions, then that idea must be rejected or modified. You can see that practical work therefore plays a crucial role in the progress of science. Indeed, in science, experiment is the ultimate judge of what is to be believed and what is not. No matter how pleased you might be with a theory or idea, or how elegant it appears to be, if the results do not bear out its predictions, it must be consigned to the scrap heap!

This has frequently occurred throughout the history of science. One celebrated example is that of the movement of blood within the body. In 1628, William Harvey, an English doctor working in Oxford, made a discovery of great significance. He discovered that blood is perpetually *circulated* throughout the

body. The existence of a pumping heart and blood enclosed in vessels had been known for centuries but up to that time the traditional theory was that the flow of blood within the vessels was reversible, surging in one direction and then in the other. Interestingly, Harvey did not set out to make a revolution; it emerged from the logic of his scientific work. He conducted original investigations including dissections of human cadavers and also large numbers of animals. In the course of his investigations, Harvey came, via observations and experiments, to a conclusion that was at variance with the commonly accepted theory.

The process of practical work leading to theories and more practical work or experiments is called the scientific method. Its great strengths are its thoroughness together with its ability to constantly adapt or modify our understanding of the natural world. Thus no theory is set in concrete, but must be continuously reviewed in the light of new data and new findings.

You should also note that the basic approach is always the same whatever branch of science is involved, and whatever method of investigation or type of apparatus is used.

Now let us concentrate on the important aspects of carrying out a scientific investigation—making observations and measurements and then recording them.

1.2 QUALITATIVE AND QUANTITATIVE MEASUREMENTS

Initially observations are likely to be **qualitative**; this means that they do not involve any collection of numerical data. A qualitative measurement involves making observations with your senses. For example, when milk is placed in a saucepan and heated you might observe that:

it gets hot—your hand held near the saucepan *feels* the heat

it boils—you *see* bubbles

it froths—you *see* it rising in the saucepan.

Such observations are very useful as a starting-point. However, to enable us to develop a theory (for example that milk always boils at the same temperature), **quantitative** measurements or observations need to be made, that is measurements with numbers or figures attached to them. Measurements can be made using very simple instruments, such as a ruler or thermometer, or very complex ones that make use of sophisticated electronic technology, such as an electron microscope or laser.

But no matter what the means of measurement the same basic approach is used. You must:

(a) plan what you want to achieve, that is the *aim* of the practical work

(b) decide on the units of measurement to be used

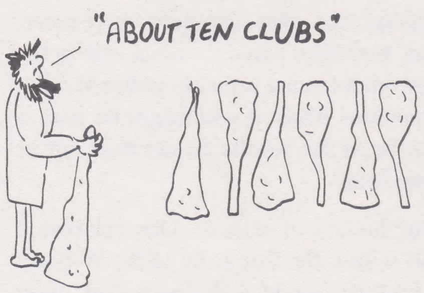
(c) decide how the measurements are to be taken

(d) devise a method for recording and presenting the results

(e) decide on the accuracy required.

The following sections consider each of these points in turn using a straightforward example of measuring the size of sheets of card for illustration. You will not have to take measurements in this Exercise since all of the data is provided. Once completed, you will use these ideas for yourself, in an investigation of the dimensions of leaves. In this Exercise you will need to collect your own results.

Sections 2 to 4 consider an Exercise to investigate the sizes of sheets of card. Suppose you are a paper supplier. You are asked by a potential customer what sizes of sheets of card you have in stock. He wants to use the card to make calendars and is concerned to have the minimum amount of waste material and, at the same time, to cut the sheets of card so that the calendars fit standard sized envelopes. You therefore have to measure the sheets of card.



Quantitative measurements

2 THE SIZES OF SHEETS OF CARD

Let us look into this imaginary investigation by working through items (a) to (e) above. You may be surprised at the number of points you need to consider when carrying out such an apparently straightforward exercise!

2.1 THE AIM OF THE PRACTICAL WORK

When carrying out practical work you need to be clear about the aim of the investigation, that is what you are trying to achieve or find out.

□ Clearly state the aim of the exercise on card.

■ The aim is to check your stocks of card, by measurement, to see whether they are suitable for the potential customer's requirements.

2.2 UNITS OF MEASUREMENT

Decide on the units of measurement

Moving on to the next point of the investigation, you need to decide on the units of measurement. Being clear about the units of measurement you are using is very important. Without them, results of investigations and experiments would be meaningless.

Consider the following example: imagine a weather forecast that just gives the temperature as 30 degrees.

□ Is this a hot or cold day?

■ As you have not been told whether the temperature is being measured in degrees Celsius (°C) or degrees Fahrenheit (°F), you do not know whether 30 degrees means:

30 °F—just below freezing

or 30 °C—a very hot day.

It is equally unhelpful to be told that the length of a piece of material is 10; there is a considerable difference between, say, 10 centimetres and 10 metres.

In science the units used are **SI units**. This may sound a bit mysterious, but in fact SI stands for 'Système International d'Unités'. In 1960 an international conference formally approved a set of metric units as standard, so replacing the many different national systems of measurement that had been used in science up to that point.

□ What is the advantage of using SI units?

■ The advantage is that if everyone is using the same units world-wide, there is no need to convert laboriously from one system to another in order to compare results in different countries.

In everyday life you may buy your milk in pints, and measure length in miles, or feet and inches. However, when it comes to scientific measurements, you should *always* use SI units. Gradually more and more metric units are being adopted in the United Kingdom, and the result is many everyday units are now similar to SI units.

The standard SI units for:

length is abbreviated to m

time is seconds, abbreviated to s

mass is kilogram, abbreviated to kg.

A more complete list of SI units is on the back cover of these Modules for you to refer to as you progress through the course.

The use of SI units raises an important question. Does their use mean that measurements of length, for example, should always be in metres? (One metre is approximately the length of an adult's stride.) Think about measuring the distance from the Earth to the Moon, or the size of a pin head! To say that the Moon is 384 321 000 metres is cumbersome. Similarly to give the size of a pin head as being 0.001 metres in diameter is unwieldy. It is often *more* convenient to make the measurement in larger or smaller multiples of the metre. For example, large distances are measured in kilometres (km) and small distances in millimetres (mm). The prefix *kilo* means 'one thousand of' so there are 1000 metres. in a kilometre. The prefix *milli* means 'one thousandth of', so a millimetre is one thousandth of a metre. This relationship between km, m and mm can be summarized as follows:

km	m	mm
1	1 000	
	1	1 000

Although you might measure in kilometres and millimetres you may need to convert them to metres.

□ Suppose the distance from Birmingham to Newport Pagnell is 110 kilometres. How many metres is this?

■ 110 000 m

$$110\,\text{km} = 110 \times 1\,000\,\text{m} = 110\,000\,\text{m}.$$

Box 1 shows you how to do this on your calculator.

BOX 1

Press	110	(i.e. press 1, then 1, then 0)
Press	×	
Press	1 000	
Press	=	(110 000 should appear)

□ Suppose the diameter of a 2p coin is 25 millimetres. Convert this to metres.

■ 0.025 m

$$25\,\text{mm} = \frac{25}{1\,000}\,\text{m} = 0.025\,\text{m}$$

On your calculator you would follow the steps shown in Box 2.

BOX 2

Press	25	(i.e. press 2, then 5)
Press	÷	
Press	1 000	
Press	=	(0.025 should appear)

Now you are aware of the importance of units lets consider which units are the most suitable for measuring the sheets of card.

□ Which units would you use?

■ The most convenient would be mm.

You can practise the conversion of units by doing the following SAQs. When writing down your answers do not forget to give the abbreviations for the required units of measurement!

SAQ 1 Look at the relationship given above, between km, m and mm. Fill in the gaps by converting 1 km into mm and 1 000 mm into km.

SAQ 2

(a) The distance from Cardiff to Abergavenny is 55 000 metres. How many kilometres is this?

(b) The size of the eye of a needle is 2 mm. Convert this to metres.

(c) The size of a nail head is 0.01 metres. How many millimetres is it?

3 HOW THE MEASUREMENTS ARE TAKEN AND THE RESULTS PRESENTED

So far we have decided to work in millimetres. but how will the measurements be made (point (c) on p. 2) and, having done this, how will the results be presented (point (d) on p. 2)? Let us go on to consider these two points in detail.

3.1 TAKING THE MEASUREMENTS

Deciding on the practical method of how the measurements of the sheets of card are to taken is the next stage. In fact—as you might guess—we will decide on an ordinary plastic ruler. Had we wanted to measure the distance to the moon or the size of a pin head we may have decided to use more sophisticated equipment!

But we need to consider in more detail how the measurements of each sheet of card are to be taken. Let us now take a reading. Look at Figure 1 which shows part of a sheet of card with a ruler laid against its length.

□ What length is the piece of card?

■ The length is between 86 and 87 mm.

By estimating the nearest millimetre division to the end of the card for each measurement, we can 'round up' or 'round down' each reading to the nearest millimetre. The piece shown in Figure 1 is closer to 86 mm, so to the nearest millimetre we estimate the length to be 86 mm.

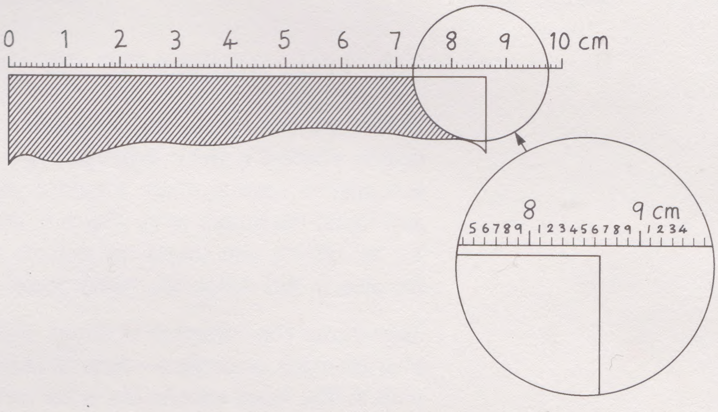


FIGURE 1 The length of a piece of card is between 86 and 87 mm.

3.2 PRESENTING DATA IN A TABLE

Having obtained some measurements the next stage in the investigation is to devise a method for recording and presenting the data. This depends upon the type and quantity of results and whether you need to carry out any calculations with them. Very frequently scientists enter their data in tables. Lets look at this further.

Recording and presenting results in a table enables you to collect the data together and present them in such a way that they are understandable to others and easy to read.

□ How many different measurements would there be for each sheet of card?

■ Two measurements, the length and the width.

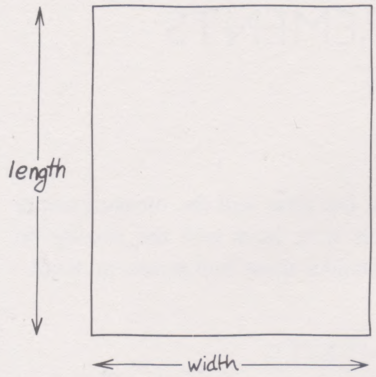


FIGURE 2 A sheet of card showing the positions at which measurements were made.

TABLE 1

Length	Width
296 mm	210 mm
227 mm	178 mm

TABLE 2 The dimensions of sheets of card

Length/mm	Width/mm
296	210
227	178

Figure 2 shows a sheet of card with the positions at which measurements were made clearly marked. Note how much easier it is to show this in a diagram than trying to describe it in words! It is often helpful to include sketch diagrams like the one shown in Figure 2 when recording and presenting practical work.

Here are some measurements that we took of two sheets of card of different dimensions.

RESULTS

296 mm by 210 mm

227 mm by 178 mm

Table 1 shows these results in table form under the headings length and width.

Notice that the units used are given *in* the Table. Can you think of a better arrangement? It is better to enter the data as simple numbers without having to attach the units, so a special convention is used for the headings of tables. The name of the quantity being measured such as length is given in each column, followed by a slash (/) and the unit of measurement (mm). Table 2 shows the *correct* way to display the data.

You may find it helpful to think of the '/' as a 'divide' sign. Since distance is measured in units of millimetres, then 'dividing' by millimetres means that you can record your measurements in the table simply as numbers.

One final and important point about a table is that it should indicate what the data relates to. This can be included in the title, as shown for the completed Table 2.

3.3 THE AVERAGE OF A SET OF READINGS

Before we can decide on the precise form of the table or tables we need to decide whether a given dimension of a sheet of cardboard should be measured just once or several times. Suppose you measured a piece of card with a ruler and found the length to be 296 mm. How sure could you be that it is 296 mm? To be certain you might measure it again and find this time it is 297 mm. Because of this difference you measure it again and find it is 295.5 mm.

This shows you a number of things. Firstly, it suggests that no measurement can ever be exact or perfect—there is always a degree of **uncertainty** associated with it. The word uncertainty is the technical term used by scientists to indicate inexactness. An old term which means the same thing is error. You should use the new term. Uncertainties are discussed in more detail in Section 5.

The differences between successive measurements suggests that it is often desirable to (i) take several readings of the *same* quantity and then to (ii) calculate the **average** of these readings to obtain the result nearest to the true value of the quantities.

But what exactly is an average? Again, you can draw on your everyday experience of this concept, since we often talk of the average age of a group of children, or the average temperature for the month of June.

> To obtain the average of a set of data, add all the readings together and divide the total by the number of readings taken.

☐ The diameter of a marble was measured four times and the results were 26 mm, 25 mm, 26 mm, and 27 mm. Calculate the average.

■ average diameter

$$= \frac{26\,\text{mm} + 25\,\text{mm} + 26\,\text{mm} + 27\,\text{mm}}{4}$$

$$= \frac{104\,\text{mm}}{4}$$

$$= 26\,\text{mm}$$

You should be able to perform this calculation on your calculator using the procedures given in Module 1.

3.4 THE FINAL FORM OF THE RESULTS TABLE

Now that you understand the need to take several readings and find the average of these readings, you are in a position to think about the final form of the results table in this card measuring exercise. How would you present the data obtained from several measurements of the lengths and widths of two sheets of card?

Tables 3 and 4 show one way to set out a table giving several readings of the same quantity as well as the average. The measurements refer to the lengths (Table 3) and widths (Table 4) of two different sheets of card, 1 and 2. You may have thought of another way, for example combining the width and length measurements into one large table.

You should always list all the individual readings, measurements or **raw data** in your table, and not just the average. Raw data means the actual measurements taken, before any calculations have been performed on them.

TABLE 3 Lengths of two sheets of card

Card	Length/mm				Average Length/mm
sheet 1	296	296	297	296	
sheet 2	227	227	228	228	

TABLE 4 Measurements of the width of two sheets of card

Card	Width/mm				Average Width/mm
sheet 1	210	209	210	209	
sheet 2	178	178	179	179	

Notice that we have also included some data in both Tables for you to work with.

□ For each row in Tables 3 and 4 calculate the average using your calculator.

■ Average length of sheet 1 is 296.25 mm
Average length of sheet 2 is 227.5 mm
Average width of sheet 1 is 209.5 mm
Average width of sheet 2 is 178.5 mm

Before entering these figures in Tables 3 and 4 we need to consider them in more detail. For example, should we give the answers to one or two decimal places given that the sheets of card were measured to the nearest whole mm? This question is about the accuracy of the results. The next section goes on to discuss the accuracy of measurements, after which you will be better informed to complete Tables 3 and 4 and enter the averages.

You can practise working out averages by doing the following SAQ.

SAQ 3

(a) The maximum daytime temperature readings for a week were as follows:

17 °C 18 °C 20 °C 18 °C 24 °C 21 °C 22 °C

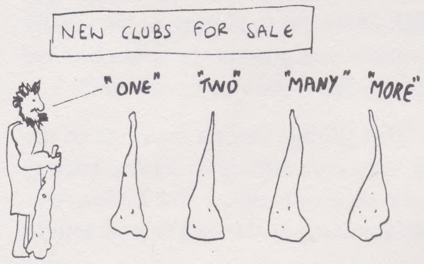
What was the average maximum daytime temperature for that week?

(b) In order to obtain an accurate value for the height of a girl, eight separate measurements were made as follows:

1.52 m 1.53 m 1.51 m 1.52 m 1.53 m 1.51 m 1.53 m 1.51 m

Calculate the average measurement of her height.

4 ACCURACY AND UNCERTAINTY



Accuracy is all important

Earlier in the Module you were introduced to the ideas of uncertainty and averages. This Section looks at the accuracy of measurements in more detail. This is the last of our list of points (given on p. 2) for us to consider in the card measuring exercise.

You probably realize by now that no measurement can ever be *exact* or *perfect*—there will always be a degree of uncertainty associated with it.

It is important to understand that uncertainty does not imply any mistake or element of bad measurement on the part of the worker. It is simply an inevitable consequence of the process of measurement.

So whenever you measure a quantity in science, you should always be concerned about the *accuracy* or *precision* of the measurement, as well as its actual value. Thus when expressing the value of a quantity, you must make some comment about the uncertainties as well.

It is possible to divide uncertainties into two broad types, depending on how they arise.

RANDOM UNCERTAINTIES OR ERRORS

Random uncertainties are so called because they occur as a result of the random nature of the measurement process itself. Thus, if you repeat the measurement of a particular quantity several times, you will usually get slightly different results on each occasion as in the card measuring exercise. This produces a spread of readings on *either side of the true value* (because random uncertainties are equally likely to result in readings that are too high or too low).

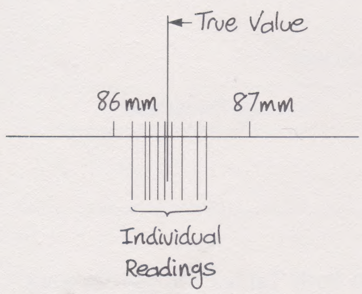


FIGURE 3 Random uncertainties produces individual readings that are clustered about the true value.

Suppose in the example shown in Figure 1 we had tried to take the readings more accurately, by attempting to measure to the nearest one-tenth of a millimetre, rather than to the nearest millimetre. Mistakes in the measurements that are due mainly to random uncertainties such as the difficulty in lining up the edge of the paper exactly with the ruler, would give readings that are spread more or less equally on either side of the true value. This is shown in the much enlarged scale of Figure 3. Note that you would not usually be able to measure to this accuracy with the naked eye; you would need a magnifying glass or some more sophisticated piece of equipment to achieve such accuracy.

Because the spread of readings is due to the effects of random uncertainties it is possible to obtain a more accurate value by taking the average of a number of individual readings—as you learned in the preceding section. This average will be much closer to the true value than any individual reading is likely to be.

SYSTEMATIC UNCERTAINTIES OR ERRORS

Systematic uncertainties are quite different from random uncertainties in their origin and behaviour. They arise as a result of faults or deficiencies in the measuring instrument. Here are two examples:

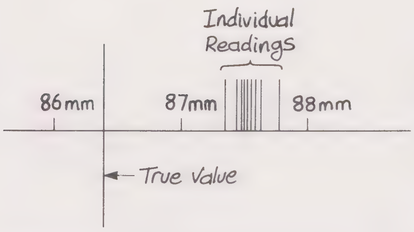


FIGURE 4 The individual readings obtained by using a ruler that has shrunk; note that they are all higher than the true value.

1 Suppose the graduations on a wooden ruler are all too close because the wood has warped and shrunk. All readings taken with such a ruler would be falsely high.

2 Suppose a stop-watch is running 'slow' compared with true time. Any measurement of time made with this stop-watch would give falsely low readings.

In both instances, unless you were aware of the faults in the measuring instrument you would be fooled into thinking that your measurements were more accurate than they really were; this is why systematic uncertainties are so insidious. Such uncertainties systematically *shift* all the measurements in one direction. This shift can either make all the readings systematically higher than the true value, as in the case of the ruler in our example, or they can make them all systematically lower, as in the case of the stop-watch.

The only successful ways to eliminate systematic uncertainties are either to be aware of them and make suitable corrections, or (better) to replace the faulty equipment!

Figure 4 shows the effect on our card measuring exercise of using a ruler that had shrunk due to warping.

□ Why do you think there is a spread of values in the readings in Figure 4, as well as a systematic shift to higher values?

■ Because there are random uncertainties present in the measurement, in addition to the systematic uncertainties.

Having measured a quantity, or worked out its value on the basis of some measurements, how should you quote the result bearing in mind the uncertainties attached to it? This is the question considered in the next section.

4.1 INTRODUCING SIGNIFICANT FIGURES

It may come as a surprise to you that the number of figures you write down when you are quoting data, or presenting the result of a calculation based on some data, is not arbitrary. Whether you write 26 mm

or 26.0 mm
or 26.00 mm,

reflects the accuracy to which the value is known. This is an extremely important point, and it is related to the concept of **significant figures**.

In a way, you have already been using this idea. If you look back at the example of the measurement in Figure 1 (p. 5), you can see that length of paper was quoted as being 86 mm, implying that the length is closer to 86 mm than it is to 85 mm or 87 mm. In other words, we were confident that the value was between 85.5 mm and 86.5 mm. A value lower than 85.5 mm, or a value higher than 86.5 mm, would have put it closer to 85 mm or 87 mm respectively. So, by writing length as 86 mm, we are implying an accuracy within plus or minus 0.5 mm, which is usually written as ± 0.5 mm.

In this case a measurement of 86 mm is said to be given to two significant figures. The number of significant figures is set by the accuracy with which measurements were made with the ruler in question.

How do you know the number of significant figures to use for the value of a measured quantity? In general, the number of significant figures used is the number of accurately known digits, plus one uncertain digit. So in the above example, the number 8 is known accurately, and the number 6 is uncertain, giving a total of two significant figures.

A further point concerns the number of digits displayed on your calculator. Suppose your calculator displays the following set of figures:

2.170 45

You could write this as:

2	(to one significant figure)
2.2	(to two significant figures)
2.17	(to three significant figures)
2.170	(to four significant figures)

Let us look at these examples more closely. We have used the convention that if the last figure is followed by a number from 0 to 4, it is unchanged, but if it is followed by a number from 5 to 9, it is rounded up. This is the convention that you should use from now on.

☐ Give 2.170 45 to five significant figures.

■ 2.170 5

You should appreciate that there is a difference between say, 2.17 and 2.170, even though the numerical values appear to be the same.

☐ Can you say what the difference is?

■ In the case of 2.17 (to three significant figures), it is the number 7 that is uncertain. In 2.170, the number 7 is known accurately, and it is 0 that is now uncertain.

You may have to perform calculations involving quantities that are known to different degrees of accuracy, such as

$$507.1\,\text{mm} + 190.28\,\text{mm} = 697.38\,\text{mm}$$

In this case you should always give the final result to the same number of significant figures as the *most* uncertain quantity in the calculation. You need to remember this because your calculator will not! Most calculators will give an answer to as many figures as they can display.

☐ Give the answer for the above addition with the correct number of significant figures.

■ 697.4 mm. (The answer is given to 4 significant figures because of the uncertainty of the digit 1 in 507.1)

The two main points to remember about significant figures can be summarized as follows:

> The number of significant figures for the value of a measurement, is the number of accurately known digits plus one uncertain digit.

> If two or more quantities are combined (e.g. by adding or dividing one by another), then the result should be quoted only to the same number of significant figures as the most *uncertain* quantity.

One final point about significant figures. The value 0.02 is a number to one significant figure (but two decimal places). This is because the zeros *before* the 2 do not count as significant figures.

☐ Write the following number to two significant figures 0.031 67

■ 0.032 (the 1 is rounded up and the zeros before the 3 do not count).

4.2 ACCURACY OF THE RESULTS

Having considered uncertainties and significant figures let us now return to the average values for the sizes of the 2 sheets of card on p. 7 so that Tables 3 and 4 can be completed.

□ Look back at your results for the averages of the lengths and widths of the sheets of card on p. 7. To how many significant figures should you give your answers?

■ To three significant figures, corresponding to the approximate accuracy of the readings.

□ Give the averages and uncertainties for the data in Tables 3 and 4.

■

Length of sheet 1	296 mm ± 0.5 mm
Length of sheet 2	228 mm ± 0.5 mm
Width of sheet 1	210 mm ± 0.5 mm
Width of sheet 2	179 mm ± 0.5 mm

If you did not get the answer to this ITQ correct you may need to check your understanding of significant figures and uncertainties.

4.3 CONCLUSIONS OF THE INVESTIGATION

When carrying out the practical exercise to measure the sheets of card we followed a five point approach, which is given on p. 2. This Section summarizes these five points in turn.

(a) The original aim of the investigation was to check the dimensions of sheets of card that you had in stock.

(b) Next we decided on the units of measurement to use. We decided that mm would be the most appropriate.

(c) We discussed how the measurements were to be taken and we decided to use a plastic ruler. Because of the uncertainties of the measurements we took multiple readings of each dimension and then worked out the average.

(d) The method we chose for recording and presenting the data was to use tables. The lay out of a table depends on the number of different measurements made and whether any calculations have to be made using them, such as working out averages.

(e) We considered the accuracy of the measurements. Related to this are (i) the uncertainties in the process of measuring and (ii) the number of significant figures to which the results should be quoted.

Conclusions can be drawn from results of practical work. These should relate to the original aim of the investigation. We can conclude that the dimensions of the sheets of card are of the following sizes and uncertainties.

Sheet 1 296 mm ± 0.5 mm by 210 mm ± 0.5 mm
Sheet 2 228 mm ± 0.5 mm by 179 mm ± 0.5 mm

SAQ 4 Write the following quantities to two significant figures:

(a) 2.09 (b) 2.04 (c) 0.01 (d) 9.97

SAQ 5 In order to work out the speed of something such as a car, the distance covered is divided by the time taken:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Thus the units of speed are those of distance over time, such as kilometres per second. Use your calculator to work out the speeds (in kilometres per second, km/s), *to an appropriate number of significant figures*, corresponding to the following data:

(a) a train covering a distance of 1 km in 30 s

(b) a car covering a distance of 1.20 km in 76 s

(c) a lorry covering a distance of 0.27 m in 0.01 s

SAQ 6 Refer back to the height measurements in SAQ 3(b) on p. 8 Suppose that one further measurement of 1.58 m was made, in addition to those given in the question. Re-calculate the average height, expressing your result to an appropriate number of significant figures.

5 FRACTIONS, RATIOS AND PERCENTAGES

This Section introduces some concepts which are very useful in expressing the relationships between sizes of various quantities.

5.1 FRACTIONS AND RATIOS

The term fraction means that a quantity is a part of a whole, and is the result of separating a whole amount into a number of equal parts. We often use fractions in our everyday lives. Indeed, you have already met some simple fractions and their decimal equivalents in Module 1. Here are some more useful points about fractions.

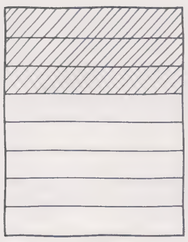


FIGURE 5 A rectangular area divided into 3/8 and 5/8 fractions.

Look at Figure 5 which shows a rectangular area such as a piece of paper, divided up into *eight* equal parts, three of which are shaded, and five are unshaded. Here three-eighths (written 3/8) is the fraction of the whole that is shaded, and five-eighths (5/8) is the unshaded fraction.

Note that fractions can be written in two different ways:

five-eighths can be written as 5/8 or $\frac{5}{8}$

The first of these is more convenient for text, while the second is better for calculations—as you will see below. Both will be used in *Into Science*.

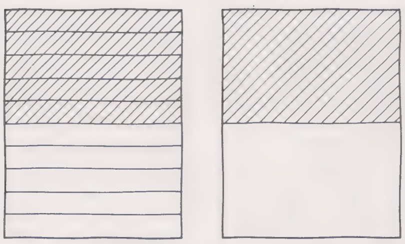


FIGURE 6 Two rectangular areas. (a) is divided into two fractions of 5/10 each and (b) is divided into two fractions of 1/2 each.

It is sometimes possible to express the same fraction in more than one way. Look at Figure 6. You can see that both 5/10 and 1/2 represent the same fraction of the whole, and such fractions are said to be **equivalent fractions**. It is possible to change 5/10 into 1/2 by dividing both the top (called the *numerator*) and the bottom (the *denominator*) by the same amount. In the case of 5/10, we divide both by 5 to give 1/2.

When it is no longer possible to divide top and bottom by some common number, the fraction is said to be in its *lowest* terms, and fractions are normally written in this form. Try converting fractions into their lowest terms by answering the following questions.

Look again at Figure 5. Draw a vertical line down the middle of the rectangular area. The rectangle is now divided into 16 parts; these are sixteenths of the whole.

□ How many sixteenths are shaded?

■ 6/16

You already know that the shaded area is 3/8.

□ By what number do you have to divide the top and bottom of 6/16 to get 3/8?

■ By 2.

□ Is the fraction 3/8 in its lowest terms?

■ Yes, because there is not another number that will divide into both numerator and denominator.

□ 10/16 of the rectangle are unshaded. Is this fraction in its lowest terms?

■ No, both upper and lower parts can be divided by 2 to give 5/8.

The **ratio** between two (or more) similar quantities expresses the relationship that exists between them. Thus if a comprehensive school has 800 girl and 600 boy pupils, the ratio of girls to boys is written as:

800 : 600

Note the use of a colon (:) to separate the two numbers. Ratios help you to see at a glance the proportions of the numbers relative to each other.

Just as for fractions, ratios can be simplified quite considerably by dividing the numbers on both sides of the colon by a common factor. So 800 : 600 is the same as 8 : 6 (dividing both numbers by 100), or 4 : 3 (dividing again by 2). Note that ratios do not have units attached to them.

Try to convert a ratio into a fraction by doing the next ITQ

□ A certain pale pink shade of paint is produced by mixing red and white paints as follows: 3 parts red and 6 parts white. (a) Express these quantities as a ratio. (b) Work out what fraction of the total volume of paint will be red. Give the answers in the lowest terms.

■ (a) The red and white paints are used in the ratio 3 : 6. This can be simplified to 1 : 2 by dividing both by 3.

(b) There are 9 parts altogether, so that red paint makes up 3 parts out of 9, or 3/9 which can be simplified to 1/3 by dividing both top and bottom by 3.

You can test your understanding of these ideas by trying the following SAQ, which uses data of some size measurements of paper similar to those introduced earlier.

SAQ 7

(a) A sheet of A4 paper is measured to be 210 mm wide by 298 mm long. Write down the ratio of the width to the length of A4 paper. Express this ratio as a fraction in its lowest terms. Now, using a calculator, express the fraction as a decimal quantity to three significant figures.

(b) Two A5 sized sheets of paper can be obtained by folding and cutting an A4 sheet in half along a line parallel to its width, as shown in Figure 7. What is the ratio of width to length as a fraction, and as a decimal quantity to the appropriate number of significant figures.

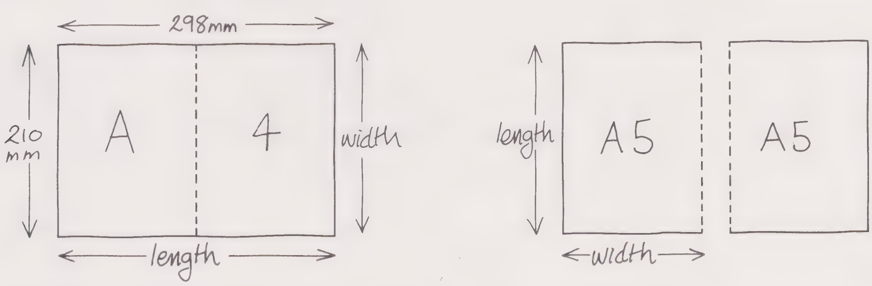


FIGURE 7 An A4 sheet of paper cut into two to give two A5 sheets.

5.2 PERCENTAGES

You will have met percentages frequently in everyday life. VAT often needs to be added to a purchase price, and bank and savings account interest is usually quoted as a percentage of the sum deposited. We call a quantity a **percentage** when the fraction of a whole is expressed in hundredths. So a half (1/2) expressed as a percentage is fifty hundredths, that is 50/100. This is written as 50% (fifty per cent).

The way to convert a fraction into a percentage is by multiplying the fraction by 100.

For example to convert one eighth into a percentage:

$$\frac{1}{8} \times 100 = \frac{100}{8} = 12.5 \text{ percent (written } 12.5\%\text{)}$$

50% of clubs are defective

The alternative situation also arises, where you need to change a percentage quantity into a fraction. To do this you divide by 100, as follows:

$$15\% = \frac{15}{100} = \frac{3}{20} \text{ (dividing top and bottom by 5 to give the lowest terms).}$$

A few calculators have a special percentage key that enables you to punch in the percentage required directly.

Try the following SAQs which test your ability to use percentages.

SAQ 8 An electric toaster normally costs £16.50 (without VAT), but is on sale at a 10% discount. If VAT at 17.5% is added on to the discounted price, calculate the amount of VAT due.

SAQ 9 Over a period of 6 weeks, the mass of a puppy increases from 0.56 kg to 0.89 kg. Express the mass increase as a percentage of the original amount. Give your answer to two significant figures.

6 MEASURING THE DIMENSIONS OF LEAVES

Now it is your turn to do some practical work involving measurements. Instead of imaginary sheets of paper you will use leaves from a bush or tree that you collect for yourself.

EXERCISE: INVESTIGATING THE DIMENSIONS OF LEAVES

For this exercise you will need:

- a 30 cm ruler divided into centimetre and millimetre divisions
- a length of cotton
- ten leaves, picked at random from a tree or bush. (This means that you should collect leaves from all over the bush or tree in a haphazard way.)

This is an exercise to measure the widths and lengths of several leaves from the bush or tree. The aim is to investigate whether the relative proportions of these dimensions are constant from leaf to leaf. As for the Exercise to measure the dimensions of sheets of card there are several points that you need to consider carefully before you begin.

You should collect a sample of 10 leaves from a convenient bush or tree—any type will do. A drawing of two types of leaf, laurel and oak are shown on p. 21 But before you begin carrying out any measurements on your leaf specimens, try answering the next SAQ to help you to focus on the questions that need considering.

SAQ 10 Make some brief notes (just one or two sentences each), outlining your thoughts on questions (a) to (e) below. When you have finished, have a look at the answers and comments given at the back of the Module before continuing with the rest of the Exercise.

(a) What is the aim of this investigation? Discover proportion between w + Ht.

(b) What units of measurement should you use? mm

(c) How are the measurements to be taken? By Ruler,

(i) Is one measurement of each dimension sufficient for each leaf? NO get an average measurement / leaf.

(ii) Where on the leaves should you make the measurements? Top or Bottom MAX. width

(d) How should you present your results? – a book detailed – tabular form

(e) How accurate should the measurements be? TO 0.5 mm.

Check your answers to this SAQ before proceeding.

PROCEDURE

Having decided on the answers to these questions, here is the order in which to proceed.

1 Carry out width and length measurements on your leaf specimens and present your results in a table or tables.

2 Express the width and length measurement of each leaf as (a) a ratio, (b) as a fraction in its lowest terms and (c) as a decimal quantity (to 2 decimal places).

3 Comment on the accuracy with which you were able to measure *individual* leaf dimensions compared with the variation in the dimensions between your specimens.

4 Comment on whether the relative proportions of width : length were constant from leaf to leaf.

5 Compare your table(s) and comments with ours given in Appendix 2.

7 PRACTICAL WORK: SOME USEFUL TIPS

This Section gives you some further advice about doing practical work. Keeping clear and accurate records of what you have done is essential. Two key tests for the quality of your practical work records are:

1 Could you, on the basis of your notes, write up a report of your practical work at a later stage—say, in six months' time?

2 Could your notes be used as clear guidelines to enable your practical work to be replicated, either by you or by someone else?

Although at this stage in your studies you do not have to meet such stringent requirements, it is a good idea, nevertheless, to cultivate a careful scientific approach to all your practical work. With this in mind, here are some points to note.

You should use a notebook that will not easily get mislaid to record the details of your practical work. A hardcover book of A4 size would be ideal. The important point is that you should *not* use loose bits of paper, shorthand dictating pads and the like; to do this is to ask for trouble!

Record your information clearly and legibly, not just neatly. Write down everything you think is important *while you are actually performing the practical work*. Also keep an accurate record of any equipment that was used.

The importance of recording raw data, together with the units of measurement used, and an estimate of the accuracy of the measurements has already been stressed.

Finally, it is useful to make a few notes on any provisional conclusions you may have drawn from the results, immediately after completing the measurements.

As we mentioned earlier, you certainly will not be expected to be able to put all this into practice immediately. Carrying out practical work is a bit like cooking or gardening. At first you forget to do some basic things, but gradually you become more and more expert at it, until you eventually develop a sort of sixth sense about the right procedures to adopt to obtain reliable and consistent results.

8 OVERVIEW

SUMMARY

These are the concepts you have learnt about in this Module:

- The difference between observations (qualitative measurements) and quantitative measurements is that the former uses your senses whereas the latter involves numerical data.
- SI units are used for scientific measurement.
- The average of a set of data is obtained by adding together all the values and dividing by the number of readings taken.
- Random uncertainties inevitably arise in the process of measuring whereas systematic uncertainties arise as a fault in an instrument.
- The number of significant figures in a quantity reflects the accuracy with which that quantity is known.

SKILLS

Now that you have completed this Module, you should be able to:

- set out data in tables
- use significant figures correctly
- express quantities as ratios, fractions and percentages
- convert units of distance between kilometres, metres and millimetres.

In addition you should also be aware of the importance of keeping clear and accurate records of experimental work, and asking yourself appropriate questions before taking readings or measurements.

APPENDIX 1: EXPLANATION OF TERMS USED

As you progress through the Modules you will meet some of these terms and concepts again and learn more about them.

AVERAGE To obtain the average of a set of data, add all the readings together and divide the total by the number of readings taken.

PERCENTAGE A percentage is a fraction of a whole expressed as a given number of hundredths; e.g., twenty-seven hundredths is the same as twenty-seven percent.

QUALITATIVE OBSERVATION A measurement not involving numerical data; thus it might be the observation of the shape, colour or smell of something. Alternatively, it could involve the description of how an object moves, or how an animal behaves.

QUANTITATIVE MEASUREMENT A measurement involving the taking and recording of numerical readings—for example, taking a reading of time with a stop-watch, or measuring a certain distance using a ruler or other instrument.

RANDOM UNCERTAINTIES Uncertainties that arise as a result of errors in the measuring process, or because of limitations in the scale on which the reading is based. For example, if you are taking a series of readings with a stop-watch, there will always be a small *random* variation in the time that you press the start and stop buttons. This variation increases the uncertainty about the accuracy of the start time. Similarly, random uncertainties can occur because of the difficulty in distinguishing between the finest divisions on a ruler or other measuring instrument.

RATIO The ratio between two similar quantities express the relationship in size that exists between them. The two numbers that specify the ratio are separated by a colon (:). A ratio can also be expressed in the form of a fraction or a decimal.

RAW DATA Readings taken down exactly as they appear on the instrument being read, without any subsequent calculations or analysis being performed on them.

SI UNITS The so-called 'Système International d'Unités' is in universal use in the scientific community. To enable scientists all over the world to make their measurements according to agreed standards, it was formally decided in 1960 to adopt international standard units that should be used for all scientific measurements. You will find a list of some of the most frequently used SI units on the back cover of these Modules.

SIGNIFICANT FIGURES The number of significant figures in a quantity reflects the accuracy with which that quantity is known. The number of accurately known digits in the value of a quantity, plus one uncertain digit, are together called the number of significant figures. If several quantities are combined (e.g. by adding or dividing), then the result should be quoted to the same number of significant figures as the most uncertain quantity.

All experimental results should be quoted to the number of significant figures consistent with the accuracy of the measurement.

SYSTEMATIC UNCERTAINTIES Uncertainties that arise as a result of an instrumental fault that is permanently present. They systematically shift all the measurements in the same direction away from the true value. Examples of faults that may give rise to systematic uncertainties are: a slow or fast running stop-watch, an incorrectly graduated ruler, or the failure to set an instrument to zero before starting to take measurements with it.

UNCERTAINTIES The technical term used to describe inexactness or errors in measurements. There are two types, random and systematic uncertainties.

APPENDIX 2: SPECIMEN ANSWER FOR EXERCISE: INVESTIGATING THE DIMENSIONS OF LEAVES

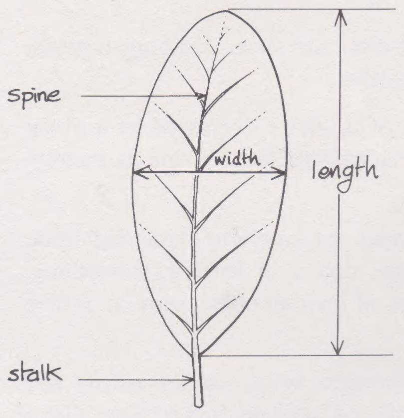


FIGURE 8 A laurel leaf showing the positions at which length and width were measured.

Before giving our results we describe the method used to carry out the measurement so that you can compare it with your own.

METHOD

Ten laurel leaves were picked at random from a bush.

The length of the leaf was measured along the spine from the point at which the leaf starts to the tip (Figure 8). The spine was not straight, so a piece of cotton was used to obtain the measurement. One end of the cotton was placed at the start of the leaf, and the cotton was laid along the spine to the tip. The cotton was then placed on a ruler and its length measured. This procedure was repeated two more times for each leaf.

You can see from Figure 8 that the width of the leaf is not constant, so it was decided to measure the maximum width. This was done by sliding a ruler up or down the leaf, keeping the zero line on one edge and noting the maximum reading for the other edge. This was repeated two more times for each leaf.

RESULTS

The results are given in Tables A and B.

TABLE A The results of the measurements of the width and length of laurel leaves

Leaf	Maximum width/mm				Length along spine/mm			
	Readings			Average	Readings			Average
1	52	52	52	52	130	131	130	130
2	52	52	53	52	145	144	144	144
3	51	51	51	51	126	126	127	126
4	41	41	41	41	100	101	101	101
5	41	41	41	41	85	84	83	84
6	38	37	37	37	97	95	96	96
7	31	31	31	31	71	73	72	72
8	24	23	23	23	54	55	54	54
9	23	24	24	24	46	46	46	46
10	28	29	29	29	47	48	47	47

TABLE B The results of measurements of laurel leaves expressed as ratios, fractions and decimals.

Leaf	Ratio of w : l	Fraction	Decimal quantity
1	52 : 130	26/65	0.40
2	52 : 144	13/36	0.36
3	51 : 126	51/126	0.40
4	41 : 101	41/101	0.41
5	41 : 84	41/84	0.49
6	37 : 96	37/96	0.39
7	31 : 72	31/72	0.43
8	23 : 54	23/54	0.43
9	24 : 46	12/23	0.52
10	29 : 47	29/47	0.62

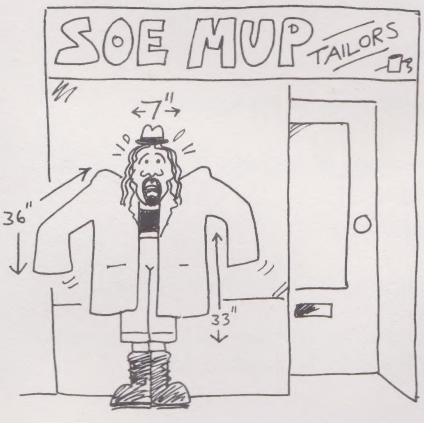
COMMENTS ON THE RESULTS

The measurements for individual leaves can be quoted to an accuracy of ± 0.5 mm.

However, it is clear that the variation in both length, (with an average of 90 mm and range of 46 mm–144 mm) and width (with an average of 38 mm and range of 23 mm–52 mm) *between different leaves* is much greater than the accuracy with which *a particular leaf* can be measured.

We can conclude that the relative proportion of width to length is not constant in the case of laurel leaves.

NOTES ON THE INVESTIGATION



How accurate should the measurements be?

1 The variation in size of leaves found in this investigation is typical of biological material. You will be aware of this sort of variability for example, from observing the range of height and foot sizes of human beings.

In general to obtain a true picture the following procedure should be adopted.

(i) take measurements from a large number of specimens

(ii) take just one measurement of width and one of length for each specimen.

One measurement of width and one of length per specimen is sufficient, since the variation between samples is much greater than the uncertainties of the measurement of each individual reading.

2 Did you find it easy to compare the ratios or fractions? With our data it is quite difficult. It is much easier to compare the ratios expressed as decimal quantities.

SAQ ANSWERS AND COMMENTS

SAQ 1 The relationship between km, m and mm.

km	m	mm
1	1 000	1 000 000
0.001	1	1 000

SAQ 2

(a) 55 km;

$$55\,000\,\text{m} = \frac{55\,000}{1\,000}\,\text{km} = 55\,\text{km}$$

(b) 0.002 m;

$$2\,\text{mm} = \frac{2}{1\,000}\,\text{m} = 0.002\,\text{m}$$

(c) 10 mm;

$$0.01\,\text{m} = 0.01 \times 1\,000\,\text{mm} = 10\,\text{mm}$$

SAQ 3

(a) 20 °C

(b) 1.52 m

SAQ 4

(a) 2.1

Here we have rounded *up*, because the last significant figure is followed by a number from 5 to 9.

(b) 2.0

Here we have rounded *down*, because the last significant figure is followed by a number from 0–4.

(c) 0.010

If you got this wrong you may have forgotten that the zeros preceding the 1 do not count as significant figures.

(d) 10

When the 9 (right of the decimal point) is rounded up (because it is followed by a 7), the 9 (left of the decimal point) becomes 10, which then represents two significant figures.

SAQ 5

(a) The speed of the train in km/s is:

$$\frac{1\,\text{km}}{30\,\text{s}} = 0.03\,\text{km/s}$$

Note that this answer is expressed to two decimal places and one significant figure, which is all that the input data justifies.

(b) The speed of the car in km/s is:

$$\frac{1.20\,\text{km}}{76\,\text{s}} = 0.016\,\text{km/s}$$

The answer is expressed to two significant figures, reflecting the accuracy of the most uncertain quantity.

(c) The speed of the lorry is:

$$\frac{0.27\,\text{m}}{0.01\,\text{s}} = 27\,\text{m/s}$$

To convert this to km/s, as required by the question, divide by 1 000, giving:

$$\frac{27}{1\,000}\,\text{km/s} = 0.027\,\text{km/s}$$

Since the input data only justifies an answer expressed to one significant figure, this should be rounded up to 0.03 km/s.

SAQ 6 The original total, for eight measurements, was 12.16 m. The new total (for nine measurements) is 13.74 m. The new average is:

$$\frac{13.74}{9}\,\text{m} = 1.526\,666\,667\,\text{m}$$

which, expressed to *three* significant figures, is 1.53 m.

SAQ 7

(a) Ratio of width to length of A4 paper is 210 : 298. This can be reduced to 105 : 149 (by dividing both numbers by 2). As a fraction this is 210/298 or, in its lowest terms, 105/149.

Expressed as a decimal quantity,

$$\frac{105}{149} = 0.705 \text{ (to three significant figures).}$$

(b) Ratio of width to length of A5 paper is 149 : 210. As a fraction, this is 149/210.

Expressed as a decimal quantity,

$$\frac{149}{210} = 0.710 \text{ (again, to three significant figures).}$$

If your answer differs from this you may not have noticed that the width of A4 paper becomes the length of A5.

SAQ 8 The value of a discount of 10% on £16.50 is given by

$$\frac{10}{100} \times £16.50 = £1.65$$

The total bill, after deducting the discount is therefore

$$£16.50 - £1.65 = £14.85$$

(Notice that this is the same as 90% of the original amount.)

You can now calculate the VAT amount by working out:

$$\frac{17.5}{100} \times £14.85 = £2.60 \text{ (rounded off to the nearest penny)}$$

SAQ 9 The mass gained by the puppy in 6 weeks is:

$$0.89\,\text{kg} - 0.56\,\text{kg} = 0.33\,\text{kg}$$

Expressed as a percentage of the original, this is:

$$\frac{0.33}{0.56} \times 100 = 59\% \text{ (to two significant figures)}$$

SAQ 10 Comments on points (a) to (e):

(a) The aim of the investigation is to determine whether the relative proportions of width and length are constant from leaf to leaf.

(b) The most suitable units of measurement are millimetres.

(c)

(i) As a general rule, more than one measurement should be taken for any reading, where possible, as a check.

(ii) Deciding exactly where you should make your measurements of the leaves is slightly more difficult than was deciding on the measuring positions for the sheets of paper. Leaves are clearly not rectangular, so in this case you need to define exactly what you mean by length and width.

Length can be defined as the length along the spine from the point at which the leaf starts to the tip. This will be the maximum length of the leaf.

Width may need to be considered in various ways, depending upon the shape of the leaf. In the case of a laurel leaf, the width increases from the stem to a maximum value and then decreases to the tip. The maximum width will simply be the widest part of the leaf (see Figure 9a).

In the case of an oak leaf, two maximum values could be given: maximum width across the leaf as a whole, and maximum width across the indented part only (see Figure 9b). So your answer to this will depend on the types of leaves you collected.

Having decided on the position of measurements you can consider how the measurements are to be taken. The spine of the leaf will not necessarily be straight; a piece of cotton laid along it and then placed against a ruler will give you your measurement of length.

You can measure the maximum width with a ruler: slide it up or down the leaf keeping the zero reading on one edge and note the variation of the reading of the other edge.

(d) Results should be presented in tabular form; a diagram could be included to show the measuring positions used.

(e) Given the method of measuring the leaves the readings will be accurate to within ± 0.5 mm.

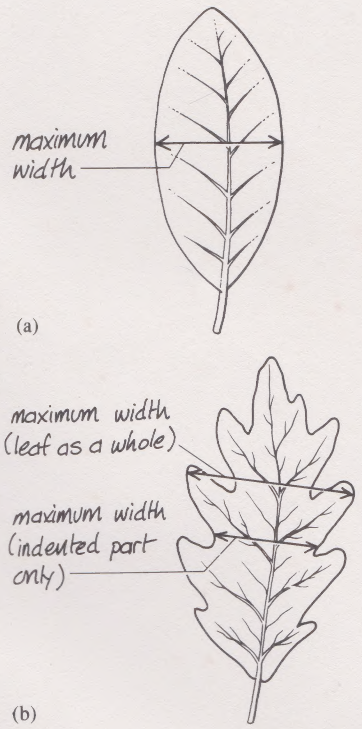


FIGURE 9 Two leaves that are shaped differently, showing possible measuring positions. (a) Laurel. (b) Oak.